Probabilistic Methods in Combinatorics

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Hints for assignment 8

Problem 1. Try to first prove such a concentration inequality for a given vertex. Then simply union bound over all choices of vertices.

Problem 2. Take a random colouring of the matrix (i.e. colour each entry of the matrix red/blue with probability 1/2, independently of the other choices), and note that if we have a sequence of row and column operations, the outcome does not depend on the order of the switches.

Problem 3.

(a) Let α be the independence number of G, and note that $\chi \geq n/\alpha$.

(b) Fix a set S of $m \geq 100 \log n$ vertices. Let X_S be the number of edges of $G[S]$. What is the distribution of X_S? Can you prove a concentration inequality?

Problem 4. Take X to be a random subset of $[n]$, and use a concentration inequality to show an upper bound on

$$\mathbb{P}\left[\left||X \cap S_i| - \frac{1}{2}|S_i|\right| > a\right],$$

where $a = \sqrt{n \log n}$.